Learning Algebraic Representation for Systematic Generalization in Abstract Reasoning

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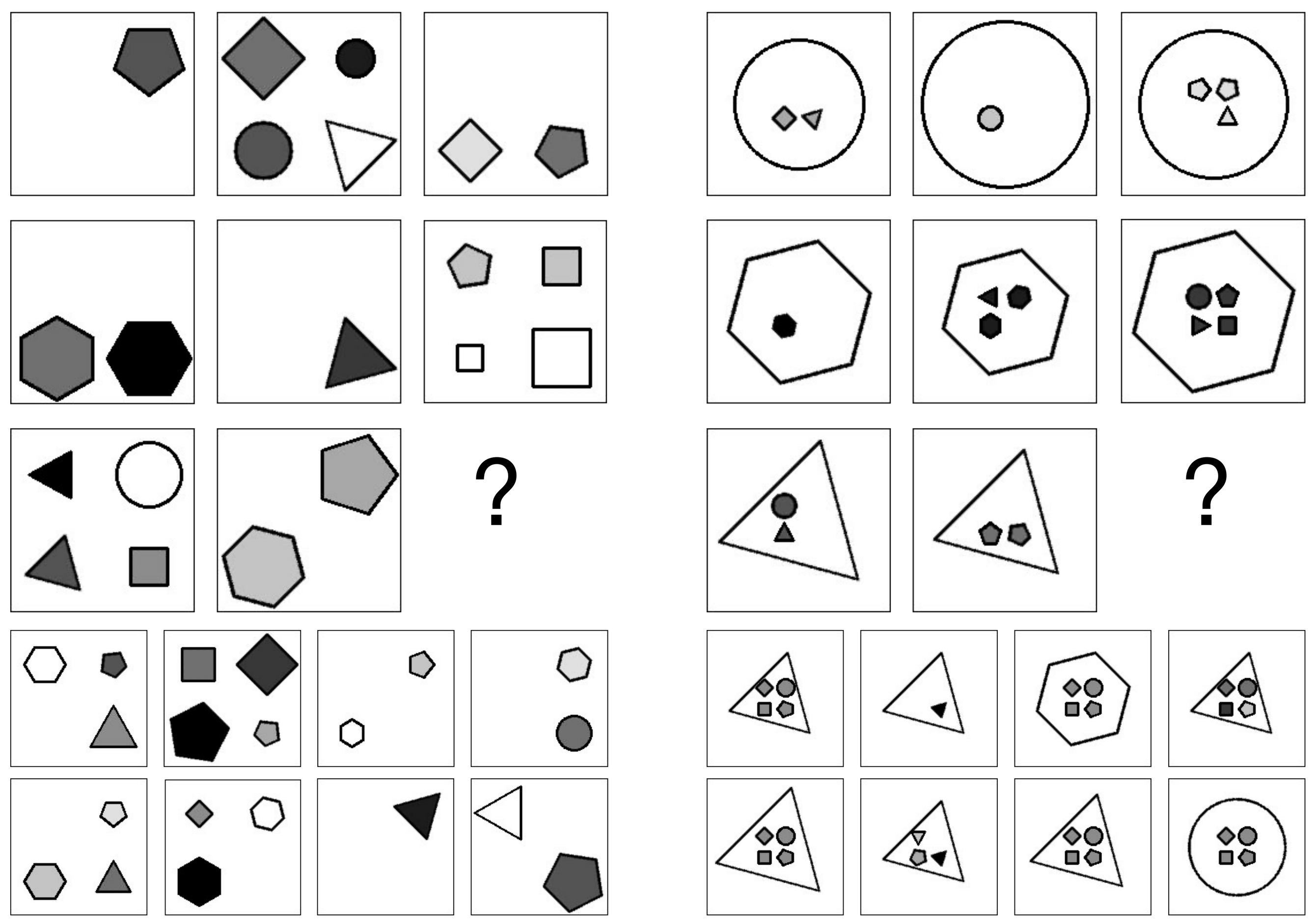
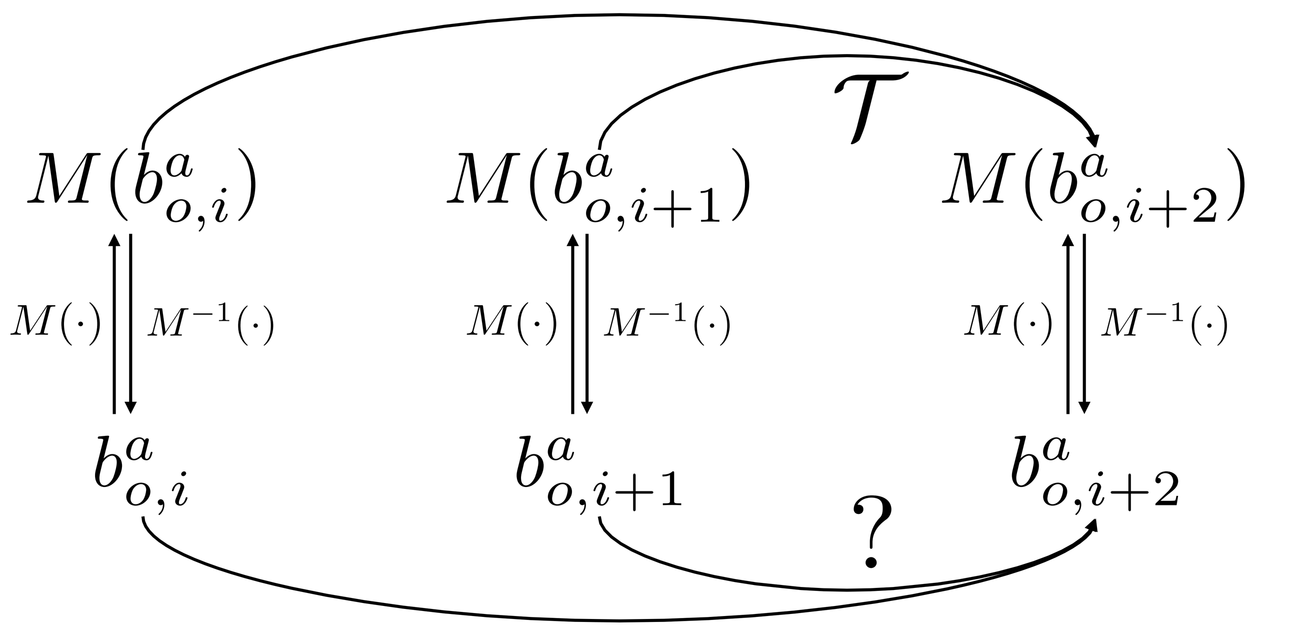
Motivation

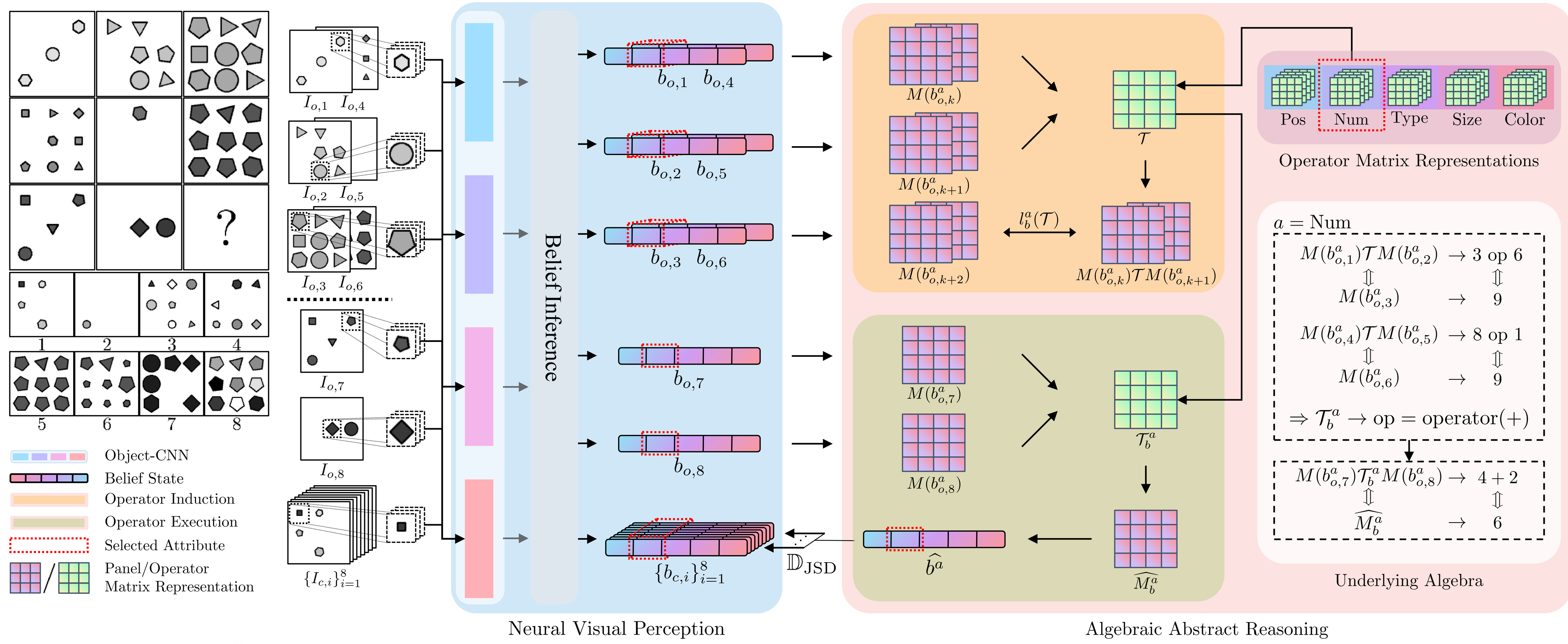


What is ***not so right*** about previous methods in solving Raven's Progressive Matrices (RPM):

- Recent endeavors propose pure connectionist models or leverage strong knowledge to circumvent the intrinsic cognitive requirement on few-shot induction of the hidden relations.
- Building on the Peano Axiom and the representation theory, we propose to learn algebraic representation that lifts discrete elements into a **matrix space** and perform relation induction by **solving inner optimization**.



ALANS



The ALANS Learner

- ALANS is a ***neuro-semi-symbolic*** method that disentangles perception and reasoning and performs relation induction on the fly, hence semi-symbolic.

Neural Visual Perception

- The design follows the PrAE learner and is composed of an object CNN and a belief inference engine.
- We use a sliding window to traverse the spatial domain of the image and feed each image region into an object CNN, which predicts object attribute distributions, *i.e.*, *objectiveness*, *type*, *size*, and *color*.
- The belief inference engine summarizes the **panel** attribute distributions by marginalizing out all **object** attribute distributions.

Algebraic Abstract Reasoning

- We encode attributes using matrix representation based on the Peano Axiom: there are two learnable matrices for each attribute, the zero matrix and the recursion matrix, to represent all elements in the attribute space $(M^a)^k M^a_0$.
- To induce a hidden relation, we perform an inner level optimization, *e.g.*, $\mathcal{T} = \arg\min \sum \mathbb{E}\left[\|M(b^a_{o,i})\mathcal{T}M(b^a_{o,i+1}) - M(b^a_{o,i+2})\|^2_F\right] + \lambda^a_b \|\mathcal{T}\|^2_F$
- After the relation is induced, we use it to predict the representation for the answer and compare it with all the candidates. The answer representation can be solved as $M = \arg\min \mathbb{E}[\|M(b^a_{o,7})\mathcal{T}^a_b M(b^a_{o,8}) - M\|^2_F]$

Performance

ALANS significantly improves over existing models on the three aspects of systematic generalization.

Method	MXGNet	ResNet+DRT	ResNet	HriNet	LEN	WReN	SCL	CoPINet	ALANS	ALANS-Ind	ALANS-V
Systematicity	20.95%	33.00%	27.35%	28.05%	40.15%	35.20%	37.35%	59.30%	**78.45%**	52.70%	93.85%
Productivity	30.40%	27.95%	27.05%	31.45%	42.30%	56.95%	51.10%	60.00%	**79.95%**	36.45%	90.20%
Localism	28.80%	24.90%	23.05%	29.70%	39.65%	38.70%	47.75%	60.10%	**80.50%**	59.80%	95.30%
Average	26.72%	28.62%	25.82%	29.73%	40.70%	43.62%	45.40%	59.80%	**79.63%**	48.65%	93.12%
Systematicity	13.35%	13.50%	14.20%	21.00%	17.40%	15.00%	24.90%	18.35%	**64.80%**	52.80%	84.85%
Productivity	14.10%	16.10%	20.70%	20.35%	19.70%	17.95%	22.20%	29.10%	**65.55%**	32.10%	86.55%
Localism	15.80%	13.85%	17.45%	24.60%	20.15%	19.70%	29.95%	31.85%	**65.90%**	50.70%	90.95%
Average	14.42%	14.48%	17.45%	21.98%	19.08%	17.55%	25.68%	26.43%	**65.42%**	45.20%	87.45%

Performance of ALANS on perception.

Object Attribute	Objectiveness	Type	Size	Color
Systematicity	100.00%	99.95%	94.65%	71.35%
Productivity	100.00%	99.97%	98.04%	77.61%
Localism	100.00%	95.65%	98.56%	80.05%
Average	100.00%	98.52%	97.08%	76.34%

Object Attribute	Objectiveness	Type	Size	Color
Systematicity	100.00%	96.34%	92.36%	63.98%
Productivity	100.00%	94.28%	97.00%	69.89%
Localism	100.00%	95.80%	98.36%	60.35%
Average	100.00%	95.47%	95.91%	64.74%

Performance of ALANS on reasoning.

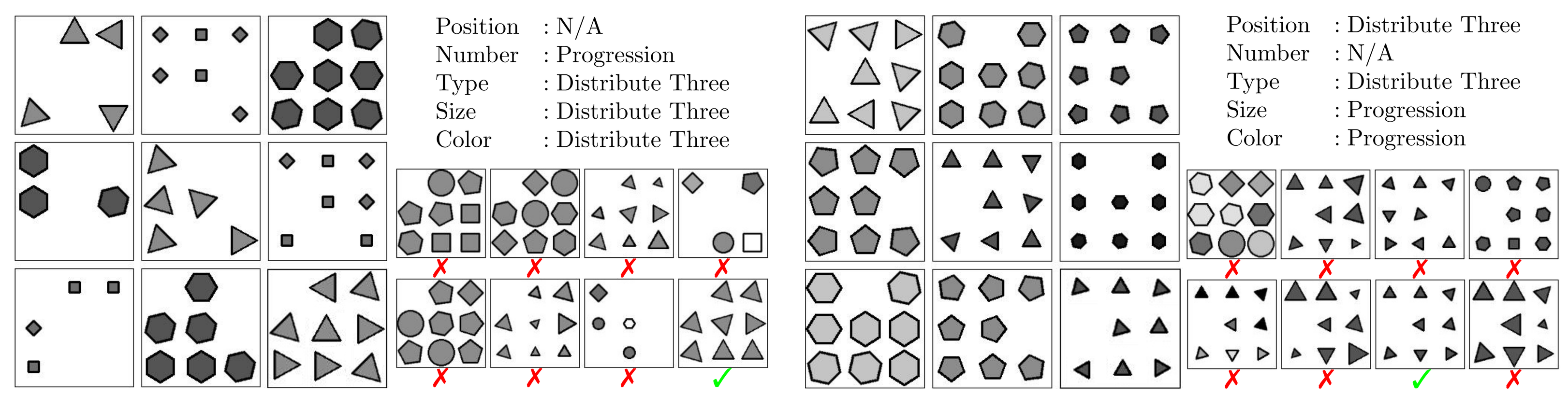
Relation on	Position	Number	Type	Size	Color
Systematicity	72.04%	82.14%	81.50%	80.80%	40.40%
Productivity	-	98.75%	89.50%	72.10%	33.95%
Localism	-	74.70%	44.25%	56.40%	54.20%
Average	72.04%	85.20%	71.75%	69.77%	42.85%

Relation on	Position	Number	Type	Size	Color
Systematicity	69.96%	80.34%	83.50%	80.85%	28.85%
Productivity	-	99.10%	87.95%	68.50%	23.10%
Localism	-	70.55%	36.65%	42.30%	33.20%
Average	69.96%	83.33%	69.37%	63.88%	28.38%

In-distribution performance of ALANS.

Method	Acc	Center	2x2Grid	3x3Grid	L-R	U-D	O-IC	O-IG
WReN	34.0%/21.5%	58.4%/24.0%	38.9%/25.0%	37.7%/20.1%	21.6%/19.7%	19.8%/19.9%	38.9%/21.3%	22.6%/20.6%
ResNet	53.4%/18.4%	52.8%/22.6%	41.9%/15.5%	44.3%/18.1%	58.8%/19.0%	60.2%/19.6%	63.2%/17.5%	53.1%/16.6%
ResNet+DRT	59.6%/20.7%	58.1%/24.2%	46.5%/18.2%	50.4%/19.8%	65.8%/22.0%	67.1%/22.1%	69.1%/21.0%	60.1%/18.1%
LEN	71.6%/32.8%	79.1%/44.8%	56.1%/27.9%	60.3%/23.9%	80.5%/34.1%	76.4%/34.4%	79.3%/35.8%	69.9%/28.5%
HriNet	45.1%/60.8%	66.1%/78.2%	40.7%/50.1%	38.0%/42.4%	44.9%/70.1%	43.2%/70.3%	47.2%/68.2%	35.8%/46.3%
MXGNet	84.0%/33.1%	94.3%/40.7%	60.5%/27.9%	64.9%/24.7%	96.6%/35.8%	96.4%/34.5%	94.1%/36.4%	81.3%/31.6%
CoPINet	91.4%/46.1%	95.1%/54.4%	77.5%/36.8%	78.9%/31.9%	**99.1%**/51.9%	**99.7%**/52.5%	**98.5%**/52.2%	91.4%/42.8%
ALANS	74.4%/78.5%	69.1%/72.3%	80.2%/79.5%	75.0%/72.9%	72.2%/79.2%	73.3%/79.6%	76.3%/85.9%	74.9%/79.9%
SCL	74.2%/80.5%	82.8%/84.6%	70.4%/79.4%	64.1%/69.9%	77.6%/82.7%	78.4%/82.6%	84.2%/87.3%	62.2%/77.2%
ALANS-V	**94.4%**/**93.5%**	**98.4%**/**98.9%**	**91.5%**/**85.0%**	**87.0%**/**83.2%**	97.3%/**90.9%**	96.4%/**98.1%**	97.3%/**99.1%**	**93.2%**/**89.5%**

Predicted results from a rendering engine.



Conclusion and Limitation

- Relations on different attributes may be related.
- The reasoning module is sensitive to perception uncertainty.
- Algebraic representation could help in formal reasoning.